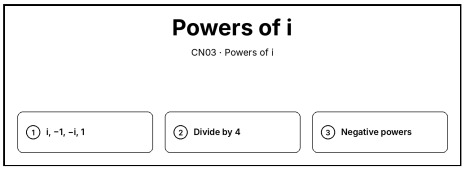
Powers of i: teacher script

CN03-V1 · CN03 Powers of i · teachers only. Without a screen, read each Say line and draw each Draw step on the board. About 93 seconds as a video.

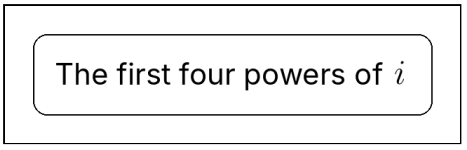
01



Say. Powers of i. The powers of i go round in fours: i, negative 1, negative i, 1, and round again. So divide the exponent by 4 and keep the remainder. And a negative power is one over a positive power.

Draw. Write the title “Powers of i” and the steps: 1. i, −1, −i, 1; 2. Divide by 4; 3. Negative powers.

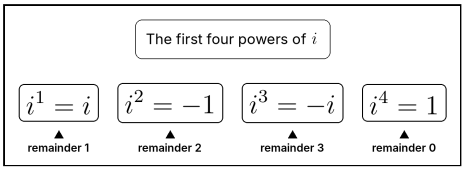
02



Say. Here are the first four powers.

Draw. Clear the board. Write “The first four powers of i” in a box at the top.

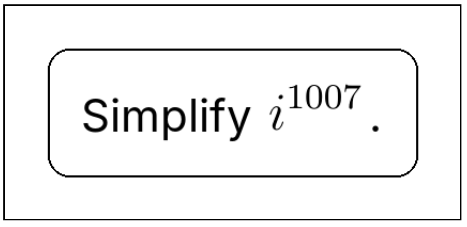
03



Say. i to the 1 is i. i squared is negative 1. i cubed is i squared times i, which is negative i. And i to the 4 is negative 1 times negative 1, which is 1. Then i to the 5 is i again.

Draw. Write $i^1 = i$ $i^2 = -1$ $i^3 = -i$ $i^4 = 1$, with a box round each term. Under $i^1 = i$, draw an arrow and write “remainder 1”. Under $i^2 = -1$, draw an arrow and write “remainder 2”. Under $i^3 = -i$, draw an arrow and write “remainder 3”. Under $i^4 = 1$, draw an arrow and write “remainder 0”.

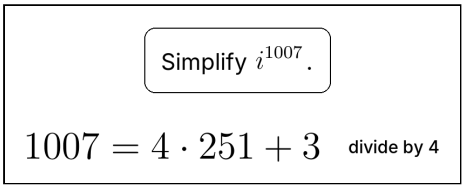
04



Say. Simplify i to the 1007.

Draw. Clear the board. Write “Simplify i^{1007}.” in a box at the top.

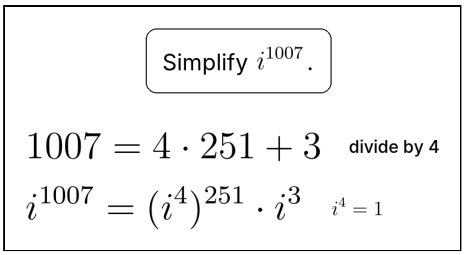
05



Say. Divide 1007 by 4: 4 times 251 is 1004, remainder 3.

Draw. Write $1007 = 4 \cdot 251 + 3$ on the next line. Beside it, write “divide by 4”.

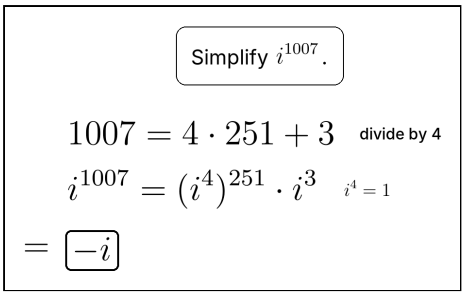
06



Say. So i to the 1007 is i to the 4, to the 251, times i cubed. And i to the 4 is 1.

Draw. Write $i^{1007} = (i^4)^{251} \cdot i^3$ on the next line. Beside it, write "$i^4 = 1$".

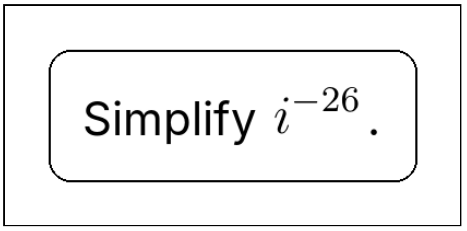
07



Say. So it is just i cubed: negative i.

Draw. Write = −i and box it on the next line.

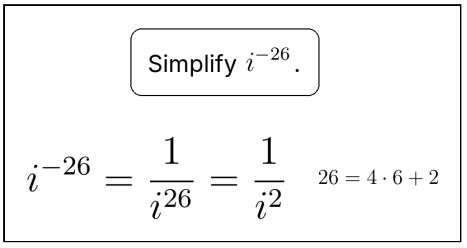
08



Say. Simplify i to the negative 26.

Draw. Clear the board. Write "Simplify i^{-26}." in a box at the top.

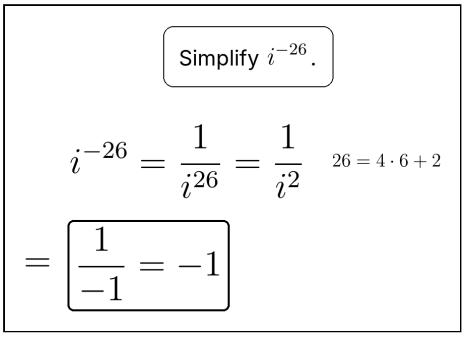
09



Say. One over i to the 26. 26 is 4 times 6, plus 2, so i to the 26 is i squared.

Draw. Write $i^{-26} = 1/i^{26} = 1/i^2$ on the next line. Beside it, write "26 = 4 · 6 + 2".

10



Say. One over negative 1 is negative 1.

Draw. Write = 1/(−1) = −1 and box it on the next line.

11

Trap i^{10}

$\times\ 10i$

✓ $i^2 = -1$

Say. Watch for the trap: i to the 10 is not 10 i. 10 is 4 times 2, remainder 2, so i to the 10 is i squared, negative 1.

Draw. Clear the board. Write “Trap” and i^{10}. Below, write 10i and cross it out ×. Then write $i^2 = -1$ with a tick ✓.

12

Your turn

Divide the exponent by 4.

Remainders 1, 2, 3, 0: i, −1, −i, 1.

Say. Now it is your turn. Divide the exponent by 4 and keep the remainder: 1, 2, 3 and 0 give i, negative 1, negative i, and 1.

Draw. Point to today’s activity: Divide the exponent by 4. Remainders 1, 2, 3, 0: i, −1, −i, 1.